

**Carnegie Mellon University**

School of Computer Science

Executive Education

## Mathematics Self-assessment

This self-assessment exercise was designed to evaluate your understanding of relevant mathematics content. A score of 80% or higher will indicate your readiness for the rigorous program material, but will not guarantee success. Should you not pass these self-assessments, we recommend you review high-quality instructional materials to help strengthen gaps and weaknesses in your core knowledge until you achieve proficiency.

## Question 1

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If  $x, y \in \mathbb{R}^+$  (i.e.  $x, y$  are positive reals) and  $a, b \in \mathbb{R}$ , choose which equations below are true. Select all options that apply.

A.  $b^{x+y} = b^x b^y$

B.  $b^{x-y} = \frac{b^x}{b^y}$

C.  $(b^x)^y = b^{xy}$

D.  $(ab)^x = a^x b^x$

### Feedback

All options are correct. All are Laws of Exponents.

## Question 2

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Assume  $\beta_0 = -1.7$ ,  $\beta_1 = 0.53$ ,  $\beta_2 = 1.03$ ,  $x_1 = 1$ ,  $x_2 = 2$

What is the output when evaluating the function?:

$$P = \frac{e^{(\beta_0 + \beta_1 x_1 + \beta_2 x_2)}}{1 + e^{(\beta_0 + \beta_1 x_1 + \beta_2 x_2)}}$$

Select the most appropriate option.

- A. 0.509
- B. 0.709
- C. 0.609
- D. 0.756

### Feedback

The correct output is 0.709.

### Question 3

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Find the limit. Select the most appropriate option.

$$\lim_{x \rightarrow -\infty} \frac{1}{x}$$

A. -1

B. 1

C.  $\infty$

D. 0

#### Feedback

The limit as x approaches negative infinity is 0.

#### Question 4

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Use the definition of a derivative to find  $df/dx$ . Select the most appropriate option.

$$f(x) = 3x^2 + 2x + 1$$

- A. 6
- B. 5
- C.  $6x+2$
- D.  $3x^2 + 1$

#### Feedback

The first derivative of  $f(x)$  is  $6x+2$ .

### Question 5

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Utilizing the derivative from above, find the minimum y-value of the following function:

$$f(x) = 3x^2 + 2x + 1$$

Select the most appropriate option.

- A. -1/3
- B. 2/3
- C. 0
- D. 1

### Feedback

The minimum of the function on the y-axis is 2/3.

## Question 6

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If  $y = 2xe^x$ , find  $\frac{dy}{dx}$

Select the most appropriate option.

- A.  $2x + 2e^x$
- B.  $2xe^x + 2e^x$
- C.  $2xe^2 + 2$
- D.  $2e^x$

### Feedback

The derivative is  $2xe^x + 2e^x$  after using the chain rule.

### Question 7

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For (column) vectors A and B,  $A = [2, 1, -1]^T$  and  $B = [3, 0, 5]^T$ . Find the solution for  $A + B$ .

Select the most appropriate option.

- A.  $[2, 0, -1]^T$
- B.  $[6, 0, 5]^T$
- C.  $[5, 1, 4]^T$
- D. No solution

#### Feedback

The sum of A and B is the vector  $[5, 1, 4]^T$ .

## Question 8

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For vectors A and B, let  $A = [2, 1]^T$  and  $B = [3, 0, 5]^T$ . Find the solution for  $A + B$ .

- A.  $[5, 1, 5]^T$
- B.  $[5, 1]^T$
- C.  $[5, 1, 4]^T$
- D. No solution

### Feedback

There is no solution because vectors of different lengths cannot be added together.

## Question 9

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Let vector  $A = [2, 1, 4]^T$ . Find the solution for  $3A$ . Select the most appropriate option.

- A.  $[6, 3, 12]^T$
- B.  $[5, 4, 7]^T$
- C.  $[6, 1, 4]^T$
- D. No solution

### Feedback

The solution for  $3A$  is the vector  $[6, 3, 12]^T$ .

## Question 10

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$$\text{Let } w = (X^T X)^{-1} X^T y$$

where:

- $w$  is a vector of weights
- $X$  is an  $n \times d$  matrix with  $n$  rows and  $d$  features
- $y$  is a column vector with  $n$  rows

Which of the following statements is true about  $w$ ?

Select the most appropriate option.

- A.  $w$  is a matrix
- B.  $w$  is an  $n \times 1$  vector
- C.  $w$  is a  $d \times 1$  vector
- D.  $w$  is a scalar

### Feedback

$w$  is a  $d \times 1$  vector.

## Question 11

$$X = \begin{bmatrix} 1 & 2 \\ 0 & 4 \\ 3 & 3 \\ 5 & -1 \end{bmatrix}$$

Let  $X = \begin{bmatrix} 1 & 2 \\ 0 & 4 \\ 3 & 3 \\ 5 & -1 \end{bmatrix}$ . Without doing the multiplication, what is  $X^T X$ ? Select the most appropriate option.

A.  $\begin{bmatrix} 1 & 0 & 3 & 5 \\ 2 & 4 & 3 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 4 \\ 3 & 3 \\ 5 & -1 \end{bmatrix}$

B.  $\begin{bmatrix} 1 & 3 & 2 & 3 \\ 0 & 5 & 4 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 4 \\ 3 & 3 \\ 5 & -1 \end{bmatrix}$

C.  $\begin{bmatrix} 1 & 2 \\ 0 & 4 \\ 3 & 3 \\ 5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 3 & 5 \\ 2 & 4 & 3 & -1 \end{bmatrix}$

D.  $\begin{bmatrix} 1 & 2 \\ 0 & 4 \\ 3 & 3 \\ 5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 2 & 3 \\ 0 & 5 & 4 & -1 \end{bmatrix}$

## Feedback

The transpose of the original matrix X multiplied by itself is represented as:

$$\begin{bmatrix} 1 & 0 & 3 & 5 \\ 2 & 4 & 3 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 4 \\ 3 & 3 \\ 5 & -1 \end{bmatrix}$$

## Question 12

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Let  $X = \begin{bmatrix} 1 & 2 \\ 0 & 4 \\ 3 & 3 \\ 5 & -1 \end{bmatrix}$ .

Which solution would yield the correct result for element  $a_{12}$  in the matrix solution of  $X^T X$ ?  
Select the most appropriate option.

- A.  $a_{12} = 1 + 0 + 9 + 25$
- B.  $a_{12} = 2 * 1 + 4 * 0 + 3 * 3 + 1 * 5$
- C.  $a_{12} = 2 * 2 + 4 * 4 + 3 * 3 + -1 * -1$
- D.  $a_{12} = 2 + 0 + 9 - 5$

### Feedback

The correct solution is:  $a_{12} = 2 + 0 + 9 - 5$ .

### Question 13

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Let  $X = \begin{bmatrix} 1 & 2 \\ 0 & 4 \\ 3 & 3 \\ 5 & -1 \end{bmatrix}$ ,  $y = \begin{bmatrix} 2 \\ 0 \\ 4 \\ 1 \end{bmatrix}$ . Find  $X^T y$ . Select the most appropriate option.

- A.  $\begin{bmatrix} 2 \\ 0 \\ 4 \\ 1 \end{bmatrix}$
- B.  $\begin{bmatrix} 2 + 0 + 12 + 5 \\ 4 + 0 + 12 - 1 \end{bmatrix}$
- C.  $[2 + 0 + 12 + 5 \quad 4 + 0 + 12 - 1]$
- D.  $\begin{bmatrix} 1 * 2 & 2 * 2 \\ 0 * 0 & 4 * 0 \\ 3 * 4 & 3 * 4 \\ 5 * 1 & -1 * 1 \end{bmatrix}$

### Feedback

$$X^T y = \begin{bmatrix} 2 + 0 + 12 + 5 \\ 4 + 0 + 12 - 1 \end{bmatrix}$$

## Question 14

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Suppose you invest \$3000 in one stock and your spouse invests \$2000 in another. Over the next year, each dollar invested in your pick will increase by  $X$  dollars and each dollar invested in your spouse's pick will increase by  $Y$  dollars.  $X$  and  $Y$  are independent random variables with the following properties:

- $X$  has a mean of 0.09 and a standard deviation of 0.20
- $Y$  has a mean of 0.12 and a standard deviation of 0.27

Your individual earnings are  $3X$  thousand, your spouse's individual earnings are  $2Y$  thousand, and your family earnings are the sum of the two earnings. What is the expected value of your family earnings in thousands?

Select the most appropriate option.

- A. 0.35
- B. 0.46
- C. 0.51
- D. 0.65

### Feedback

$$E(3X + 2Y) = 3E(X) + 2E(Y) = 3(.09) + 2(.12) = .51$$

### Question 15

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A company produces widgets in three factories, A, B, and C. Factory A produces 20% of the widgets, Factory B produces 45% of the widgets, and Factory C produces the remaining 35%. Of all widgets produced, 5% fail inspection. Of those that fail tolerance, 25% were produced in Factory A, 35% were produced in Factory B, and 40% were produced in Factory C. In Factory A, what percentage of the widgets produced failed inspection?

Select the most appropriate option.

- A. 5.7%
- B. 5.75%
- C. 6.25%
- D. 0.39%

#### Feedback

To solve, you multiply the overall rate of failure and the portion of failures in Factory A, then divide that value by the proportion produced in Factory A. This yields a value of 6.25%.

## Question 16

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We are given a biased coin with probability of heads being 0.7 ( $P(X = \text{heads}) = 0.7$ ).

We flip the coins three times and observe the outcomes heads (H), tails (T), heads (H) ( $X_1 = H, X_2 = T, X_3 = H$ ). What's the probability for this sequence of outcomes?

- A. 0.125
- B. 0.21
- C. 0.147
- D. 0.103

### Feedback

To solve, find:

$P(X_1=H \mid \theta) * P(X_2=T \mid \theta) * P(X_3=H \mid \theta)$ , or  $0.7 * 0.3 * 0.7$ , which yields a value of 0.147.

## Question 17

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The probability density function of an univariate normal (i.e. Gaussian) distribution is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2 / (2\sigma^2)}$$

Where  $\mu$ ,  $\sigma$  is the mean and standard deviation, respectively.

What is the density given the values  $x = 65$ ,  $\mu = 60$ ,  $\sigma = 2$ ?

- A.  $8.8 \times 10^{-2}$
- B.  $8.8 \times 10^{-3}$
- C.  $8.8 \times 10^{-4}$
- D.  $8.8 \times 10^{-5}$

### Feedback

The density using the given parameter values is  $8.8 \times 10^{-4}$ .

## Question 18

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Given:  $f(x, y) = x^3 y^2 + e^x + \log(y)$

What is  $\frac{df}{dx}$  and  $\frac{df}{dy}$ ?

- A.  $\frac{df}{dx} = 3x^2 y^2 + e^x$  and  $\frac{df}{dy} = 2x^3 y + \frac{1}{y}$
- B.  $\frac{df}{dy} = 3x^2 y^2 + e^x$  and  $\frac{df}{dx} = 2x^3 y + \frac{1}{y}$
- C.  $\frac{df}{dx} = 3x^2 y^2 + \frac{1}{y}$  and  $\frac{df}{dy} = 2x^3 y + e^x$
- D.  $\frac{df}{dx} = 2x^3 y + e^x$  and  $\frac{df}{dy} = 3x^2 y^2 + \frac{1}{y}$

### Feedback

$$\frac{df}{dx} = 3x^2 y^2 + e^x \text{ and } \frac{df}{dy} = 2x^3 y + \frac{1}{y}$$