

**Carnegie Mellon University**

School of Computer Science

Executive Education

## Mathematics Self-assessment

This self-assessment exercise was designed to evaluate your understanding of relevant mathematics content. A score of 80% or higher will indicate your readiness for the rigorous program material, but will not guarantee success. Should you not pass these self-assessments, we recommend you review high-quality instructional materials to help strengthen gaps and weaknesses in your core knowledge until you achieve proficiency.

## Question 1

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If  $x, y \in \mathbb{R}^+$  (i.e.  $x, y$  are positive reals) and  $a, b \in \mathbb{R}$ , choose which equations below are true. Select all options that apply.

A.  $b^{x+y} = b^x b^y$

B.  $b^{x-y} = \frac{b^x}{b^y}$

C.  $(b^x)^y = b^{xy}$

D.  $(ab)^x = a^x b^x$

## Question 2

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Assume  $\beta_0 = -1.7$ ,  $\beta_1 = 0.53$ ,  $\beta_2 = 1.03$ ,  $x_1 = 1$ ,  $x_2 = 2$

What is the output when evaluating the function?:

$$P = \frac{e^{(\beta_0 + \beta_1 x_1 + \beta_2 x_2)}}{1 + e^{(\beta_0 + \beta_1 x_1 + \beta_2 x_2)}}$$

Select the most appropriate option.

- A. 0.509
- B. 0.709
- C. 0.609
- D. 0.756

### Question 3

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Find the limit. Select the most appropriate option.

$$\lim_{x \rightarrow -\infty} \frac{1}{x}$$

A. -1

B. 1

C.  $\infty$

D. 0

## Question 4

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Use the definition of a derivative to find  $df/dx$ . Select the most appropriate option.

$$f(x) = 3x^2 + 2x + 1$$

- A. 6
- B. 5
- C.  $6x+2$
- D.  $3x^2 + 1$

### Question 5

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Utilizing the derivative from above, find the minimum y-value of the following function:

$$f(x) = 3x^2 + 2x + 1$$

Select the most appropriate option.

- A. -1/3
- B. 2/3
- C. 0
- D. 1

## Question 6

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If  $y = 2xe^x$ , find  $\frac{dy}{dx}$

Select the most appropriate option.

- A.  $2x + 2e^x$
- B.  $2xe^x + 2e^x$
- C.  $2xe^2 + 2$
- D.  $2e^x$

## Question 7

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For (column) vectors A and B,  $A = [2, 1, -1]^T$  and  $B = [3, 0, 5]^T$ . Find the solution for  $A + B$ .

Select the most appropriate option.

- A.  $[2, 0, -1]^T$
- B.  $[6, 0, 5]^T$
- C.  $[5, 1, 4]^T$
- D. No solution

## Question 8

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For vectors A and B, let  $A = [2, 1]^T$  and  $B = [3, 0, 5]^T$ . Find the solution for  $A + B$ .

- A.  $[5, 1, 5]^T$
- B.  $[5, 1]^T$
- C.  $[5, 1, 4]^T$
- D. No solution

## Question 9

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Let vector  $A = [2, 1, 4]^T$ . Find the solution for  $3A$ . Select the most appropriate option.

- A.  $[6, 3, 12]^T$
- B.  $[5, 4, 7]^T$
- C.  $[6, 1, 4]^T$
- D. No solution

## Question 10

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Let  $w = (X^T X)^{-1} X^T y$

where:

- $w$  is a vector of weights
- $X$  is an  $n \times d$  matrix with  $n$  rows and  $d$  features
- $y$  is a column vector with  $n$  rows

Which of the following statements is true about  $w$ ?

Select the most appropriate option.

- A.  $w$  is a matrix
- B.  $w$  is an  $n \times 1$  vector
- C.  $w$  is a  $d \times 1$  vector
- D.  $w$  is a scalar

## Question 11

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$$X = \begin{bmatrix} 1 & 2 \\ 0 & 4 \\ 3 & 3 \\ 5 & -1 \end{bmatrix}$$

Let  $X = \begin{bmatrix} 1 & 2 \\ 0 & 4 \\ 3 & 3 \\ 5 & -1 \end{bmatrix}$ . Without doing the multiplication, what is  $X^T X$ ? Select the most appropriate option.

A.  $\begin{bmatrix} 1 & 0 & 3 & 5 \\ 2 & 4 & 3 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 4 \\ 3 & 3 \\ 5 & -1 \end{bmatrix}$

B.  $\begin{bmatrix} 1 & 3 & 2 & 3 \\ 0 & 5 & 4 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 4 \\ 3 & 3 \\ 5 & -1 \end{bmatrix}$

C.  $\begin{bmatrix} 1 & 2 \\ 0 & 4 \\ 3 & 3 \\ 5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 3 & 5 \\ 2 & 4 & 3 & -1 \end{bmatrix}$

D.  $\begin{bmatrix} 1 & 2 \\ 0 & 4 \\ 3 & 3 \\ 5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 2 & 3 \\ 0 & 5 & 4 & -1 \end{bmatrix}$

## Question 12

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Let  $X = \begin{bmatrix} 1 & 2 \\ 0 & 4 \\ 3 & 3 \\ 5 & -1 \end{bmatrix}$ .

Which solution would yield the correct result for element  $a_{12}$  in the matrix solution of  $X^T X$ ?  
Select the most appropriate option.

- A.  $a_{12} = 1+0+9+25$
- B.  $a_{12} = 2*1+4*0+3*3+1*5$
- C.  $a_{12} = 2*2 + 4*4 + 3*3 + -1*-1$
- D.  $a_{12} = 2+0+9-5$

### Question 13

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Let  $X = \begin{bmatrix} 1 & 2 \\ 0 & 4 \\ 3 & 3 \\ 5 & -1 \end{bmatrix}$ ,  $y = \begin{bmatrix} 2 \\ 0 \\ 4 \\ 1 \end{bmatrix}$ . Find  $X^T y$ . Select the most appropriate option.

- A.  $\begin{bmatrix} 2 \\ 0 \\ 4 \\ 1 \end{bmatrix}$
- B.  $\begin{bmatrix} 2 + 0 + 12 + 5 \\ 4 + 0 + 12 - 1 \end{bmatrix}$
- C.  $[2 + 0 + 12 + 5 \quad 4 + 0 + 12 - 1]$
- D.  $\begin{bmatrix} 1 * 2 & 2 * 2 \\ 0 * 0 & 4 * 0 \\ 3 * 4 & 3 * 4 \\ 5 * 1 & -1 * 1 \end{bmatrix}$

## Question 14

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Suppose you invest \$3000 in one stock and your spouse invests \$2000 in another. Over the next year, each dollar invested in your pick will increase by  $X$  dollars and each dollar invested in your spouse's pick will increase by  $Y$  dollars.  $X$  and  $Y$  are independent random variables with the following properties:

- $X$  has a mean of 0.09 and a standard deviation of 0.20
- $Y$  has a mean of 0.12 and a standard deviation of 0.27

Your individual earnings are  $3X$  thousand, your spouse's individual earnings are  $2Y$  thousand, and your family earnings are the sum of the two earnings. What is the expected value of your family earnings in thousands?

Select the most appropriate option.

- A. 0.35
- B. 0.46
- C. 0.51
- D. 0.65

## Question 15

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A company produces widgets in three factories, A, B, and C. Factory A produces 20% of the widgets, Factory B produces 45% of the widgets, and Factory C produces the remaining 35%. Of all widgets produced, 5% fail inspection. Of those that fail tolerance, 25% were produced in Factory A, 35% were produced in Factory B, and 40% were produced in Factory C. In Factory A, what percentage of the widgets produced failed inspection?

Select the most appropriate option.

- A. 5.7%
- B. 5.75%
- C. 6.25%
- D. 0.39%

## Question 16

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We are given a biased coin with probability of heads being 0.7 ( $P(X = \text{heads}) = 0.7$ ).

We flip the coins three times and observe the outcomes heads (H), tails (T), heads (H) ( $X_1 = H, X_2 = T, X_3 = H$ ). What's the probability for this sequence of outcomes?

- A. 0.125
- B. 0.21
- C. 0.147
- D. 0.103

## Question 17

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The probability density function of an univariate normal (i.e. Gaussian) distribution is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2 / (2\sigma^2)}$$

Where  $\mu$ ,  $\sigma$  is the mean and standard deviation, respectively.

What is the density given the values  $x = 65$ ,  $\mu = 60$ ,  $\sigma = 2$ ?

- A.  $8.8 \times 10^{-2}$
- B.  $8.8 \times 10^{-3}$
- C.  $8.8 \times 10^{-4}$
- D.  $8.8 \times 10^{-5}$

Question 18

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Given:  $f(x, y) = x^3 y^2 + e^x + \log(y)$

What is  $\frac{df}{dx}$  and  $\frac{df}{dy}$ ?

- A.  $\frac{df}{dx} = 3x^2 y^2 + e^x$  and  $\frac{df}{dy} = 2x^3 y + \frac{1}{y}$
- B.  $\frac{df}{dy} = 3x^2 y^2 + e^x$  and  $\frac{df}{dx} = 2x^3 y + \frac{1}{y}$
- C.  $\frac{df}{dx} = 3x^2 y^2 + \frac{1}{y}$  and  $\frac{df}{dy} = 2x^3 y + e^x$
- D.  $\frac{df}{dx} = 2x^3 y + e^x$  and  $\frac{df}{dy} = 3x^2 y^2 + \frac{1}{y}$